2018 Fall PHYS2310 電磁學（Electromagnetism) Midterm (double sides)

[Griffiths Chs. 1-3] 2018/11/15, 10:10am – 12:00am, 教師：張存續

◇ Useful formulas $\nabla V = \frac{\partial V}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}}$

$$\nabla\cdot\mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}v_\phi$$

1. Let $\boldsymbol{\mathscr{r}} \equiv \mathbf{r}-\mathbf{r}'$ be the separation vector from a fixed point (x', y', z') to the point (x, y, z), and $\mathscr{r} = |\mathbf{r}-\mathbf{r}'|$ be its length. Show that:

   (a) $\int_V e^{-r}(\nabla\cdot\frac{\hat{\mathbf{r}}}{r^2})d\tau = ?$ and $\int_{V'} e^{-r}(\nabla'\cdot\frac{\hat{\mathbf{r}}'}{r'^2})d\tau' = ?$ (10%)

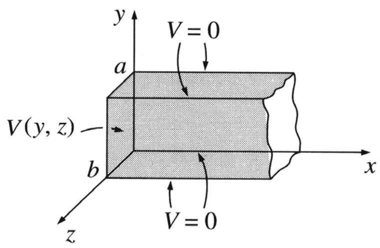
   (b) $\int_{V'} e^{-r'}(\nabla\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}^2})d\tau' = ?$ and $\int_{V'} e^{-\mathscr{r}}(\nabla'\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}^2})d\tau' = ?$ (10%)

2. An infinite long rectangular metal pipe (sides a and b) is grounded, but one end, at $x=0$, is maintained at a specific potential as indicated in the figure, $V(x=0,y,z) = V_0\sin(\pi y/a)\sin(2\pi z/b)$, where V_0 is a constant.

   (a) Write down the general solutions inside the pipe. (4%)

   (b) Find the potential inside the pipe. (10%)

   (c) Find the electric field inside the pipe. (6%)



3. A point charge q is situated at distance a from the center of a conducting sphere of radius R. The sphere is maintained at the constant potential V.

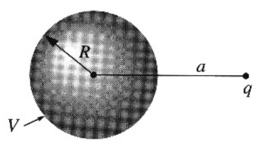
   (a) If V=0, find the position and value of the image charge. (5%)

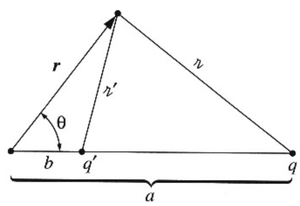
   (b) If V=V_0, find the potential outside the sphere. (5%)

   (c) Find the electric field on the surface of the metal sphere. (5%)

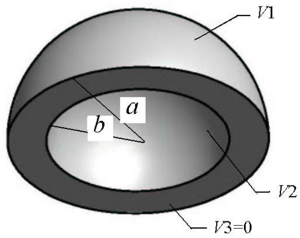
   (d) Find the surface charge density and the total charge on the metal sphere. (5%)

   [Hint: 1. use the notations shown below. 2. Assume q lays on the z-axis]





4. Suppose the potential on the surface of a hollow hemisphere is specified, as shown in the figure, where $V_1(a,\theta) = V_0(5\cos^3\theta - 3\cos\theta)$, $V_2(b,\theta) = 0$, $V_3(r,\pi/2) = 0$. V_0 is a constant.



   (a) Show the general solution in the region $b \le r \le a$. (4%)
   (b) Determine the potential in the region $b \le r \le a$, using the boundary conditions. (10%)
   (c) Calculate the electric field on the surface of the outer shell $\mathbf{E}(r = a)$. (6%)

   [Hint: $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = (3x^2 - 1)/2$, and $P_3(x) = (5x^3 - 3x)/2$.]

5. An idea electric dipole **p** is situated at the origin, and points in the *z* direction. An electric charge *q*, of mass *m*, is released from rest at a point in the *xy* plane. The potential of the dipole is $V(\mathbf{r}) = (1/4\pi\varepsilon_0)(p\cos\theta/r^2)$ and the gravitational force points in the –*z* direction.
   (a) Find the electric force between the dipole and the charge. (8%)
   (b) Find the total force (electrical and gravitational) on the charge. (6%)
   (c) Find the total potential energy. (6%)